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Research on the evaluation of public policy execution ability with picture fuzzy information

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Abstract. Public policy is one of main outputs of the political system. The realization of the government macro-regulations on development of society, economy and politics is based on effective execution of public policy. As a dynamic behavioral process, the public policy includes of lay-down, implementation and plenty of other sectors in which implementation is one of the most direct, critical and frequent activities. However, the misplay of public policy execution occurs from time to time which leads to fall-through of target of public policy, mishandle of policy and damage to government's image even has a great negative influence on social economy and political steady of a country. In this paper, we investigate the multiple attribute decision making problems with picture fuzzy information. Then, we utilize Choquet integral to propose the picture fuzzy correlated geometric (PFCG) operator. The prominent characteristic of this proposed operator are studied. Then, we have utilized the picture fuzzy correlated geometric (PFCG) operator to deal with the picture fuzzy multiple attribute decision making problems. Finally, a practical example for evaluating the public policy execution ability is given to verify the developed approach.

Keywords: Multiple attribute decision making, picture fuzzy set, picture fuzzy correlated geometric (PFCG) operator, public policy execution ability

1. Introduction

Food safety is a fundamental issue of human social development. In different historical stages of development of human society, manifested in the form of food safety issues are different. The instinctive security measures and actions of the initial human society for food safety could be seemed as the earliest human food safety policies and implementation. From passive protection to active protection, isolated measures to system measures, a religious taboo to scientific Institutional system, it is the historical background of human food safety policy implementation. The policy implementation is defined as an important variable of the policy structure and mechanism, which reflects the degree of fit between the effects of policy implementation and the expected. The particular social relations and social conflicts of the social transformation provide more targeted and restrictive platform for the studies of food safety public policy implementation, which Makes related research and discussion more focused and aggregation, from the perspective of development of human history to examine food safety, it is not difficult to find out that mankind has always had to face a wide range of food safety issues in different stages of development history [1–5]. Focus crux, as well as forms and solutions to these problems are not the same. The change and development based on the prerequisite of the historical development in essence, reflect the historic of

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food safety. Religious canons are the quite systematic social policies in the early development of human society, in which of Regulations on food safety constitute the early human society food safety policy. What people complied with religious rules was just the policy implementation. Food safety and the protection in agrarian age have a distinct era limitations. In that time, food safety policy, existed in the law as the main form, was consistent to its historical stage times and economic development. And the related policies were implemented effectively in the stable social structure. Industrialization process of human society brought food and food safety a major impact, while promoting the development of the food industry, enriching the food processing and use of human means, extending the food supply chain, Industrialization also provided a new possibility for food counterfeiting and caused more and more serious food safety problems. Historic food safety issues and the resulting limits of policy implementation is a very important fact. Any policy requirements and expectations Beyond Reality stage could not really promote effective change in food safety status. Resolving outstanding issues of food safety needs awareness and effort. Food safety and policy implementation is always a dynamic presence of a historical process, of which the ultimate outcome is depended on the direction development of history itself. Regime is one of the important material foundations for policy implementation. Many factors of food safety regulatory regime, such as regulatory approach, setting departmental functions, administrative allocation of resources, etc., directly determine the realization of policy implementation [6–10].

Atanassov [11, 12] introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the concept of fuzzy set [13]. The intuitionistic fuzzy sets have received more and more attention since its appearance [14–30]. Recently, Cuong [31] proposed picture fuzzy set (PFS) and investigated the some basic operations and properties of PFS. The picture fuzzy set is characterized by three functions expressing the degree of membership, the degree of neutral membership and the degree of non-membership. The only constraint is that the sum of the three degrees must not exceed 1 [32–43].

In this paper, we investigate the multiple attribute decision making problems with picture fuzzy information. Then, we utilize Choquet integral to propose the picture fuzzy correlated geometric (PFCG) operator. The prominent characteristic of this proposed operator are studied. Then, we have utilized the picture fuzzy correlated geometric (PFCG) operator to deal with the picture fuzzy multiple attribute decision making problems. Finally, a practical example for evaluating the public policy execution ability is given to verify the developed approach.

2. Preliminaries

Cuong [31] proposed picture fuzzy set (PFS) and investigated the some basic operations and properties of PFS.

Definition 1. [31] A picture fuzzy set (PFS) A on the universe X is an object of the form

$$A = \{\langle x, \mu_A(x), \eta_A(x), \nu_A(x)\rangle \mid x \in X\} \tag{1}$$

where $\mu_A(x) \in [0, 1]$ is called the "degree of positive membership of A ", $\eta_A(x) \in [0, 1]$ is called the "degree of neutral membership of A "and $\nu_A(x) \in [0, 1]$ is called the "degree of negative membership of A", and $\mu_A(x)$, $\eta_A(x)$, $\nu_A(x)$ satisfy the following condition: $0 \leq \mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1$, $\forall\, x \in X$.

Definition 2. [31] Given two PFEs represented by A and B on universe X, the inclusion, union, intersection and complement operations are defined as follows:

(1) $A \subseteq B$, if $\mu_A(x) \leq \mu_B(x)$, $\eta_A(x) \leq \eta_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, $\forall x \in X$;
(2) $A \cup B = \{(x, \max(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x)), \min(\nu_A(x), \nu_B(x))) \mid x \in X\}$;
(3) $A \cap B = \{(x, \min(\mu_A(x), \mu_B(x)), \max(\eta_A(x), \eta_B(x)), \max(\nu_A(x), \nu_B(x))) \mid x \in X\}$;
(4) $\bar{A} = \{(x, \nu_A(x), \eta_A(x), \mu_A(x)) \mid x \in X.\}$.

Definition 3. [21] Let $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha)$ a picture fuzzy number (PFN), a score function S of n picture fuzzy number can be represented as follows:

$$S(a) = \mu_\alpha - \nu_\alpha,\ S(\tilde{a}) \in [-1, 1] \tag{2}$$

Definition 4. [37] Let $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha)$ a picture fuzzy number (PFN), an accuracy function H of a picture fuzzy number can be represented as follows:

$$H(a) = \mu_\alpha + \eta_\alpha + \nu_\alpha,\ H(a) \in [0, 1] \tag{3}$$

to evaluate the degree of accuracy of the picture fuzzy number $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha)$, where $H(a) \in [0, 1]$. The larger the value of $H(a)$, the more the degree of accuracy of the picture fuzzy number a.

Based on the score function S and the accuracy function H, Wei [37] shall gave an order relation between two picture fuzzy number, which is defined as follows:

Definition 5. [37] Let $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha)$ and $\beta = \left(\mu_\beta, \eta_\beta, \nu_\beta\right)$ be two picture fuzzy numbers, $S(a) = \mu_\alpha - \nu_\alpha$ and $S(\beta) = \mu_\beta - \nu_\beta$ be the scores of α and β, respectively, and let $H(a) = \mu_\alpha + \eta_\alpha + \nu_\alpha$ and $H(\beta) = \mu_\beta + \eta_\beta + \nu_\beta$ be the accuracy degrees of α and β, respectively, then if $S(a) < S(\beta)$, then α is smaller than β, denoted by $\alpha < \beta$; if $S(a) = S(\beta)$, then if $H(a) = H(\beta)$, then α and β represent the same information, denoted by $\alpha = \beta$; if $H(a) < H(\beta)$, α is smaller than β, denoted by $\alpha < \beta$.

Definition 6. [37] Let $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \eta_\beta, \nu_\beta)$ be two picture fuzzy numbers, then

$$\bar{\alpha} = \alpha = (\nu_\alpha, \eta_\alpha, \mu_\alpha)$$

$$\alpha \wedge \beta = \left(\min\left\{\mu_\alpha, \mu_\beta\right\}, \max\left\{\eta_\alpha, \eta_\beta\right\}, \max\left\{\nu_\alpha, \nu_\beta\right\}\right)$$

$$\alpha \vee \beta = \left(\max\left\{\mu_\alpha, \mu_\beta\right\}, \min\left\{\eta_\alpha, \eta_\beta\right\}, \min\left\{\nu_\alpha, \nu_\beta\right\}\right)$$

$$\alpha \oplus \beta = \left(\mu_\alpha + \mu_\beta - \mu_\alpha\mu_\beta, \eta_\alpha\eta_\beta, \nu_\alpha\nu_\beta\right);$$

$$\alpha \otimes \beta = \left(\mu_\alpha\mu_\beta, \eta_\alpha + \eta_\beta - \eta_\alpha\eta_\beta, \nu_\alpha + \nu_\beta - \nu_\alpha\nu_\beta\right);$$

$$\lambda\alpha = \left(1 - (1-\mu_\alpha)^\lambda, \eta_\alpha^\lambda, \nu_\alpha^\lambda\right)$$

$$\alpha^\lambda = \left(\mu_\alpha^\lambda, 1 - (1-\eta_\alpha)^\lambda, 1 - (1-\nu_\alpha)^\lambda\right)$$

3. Picture fuzzy correlated geometric (PFCG) operator

In this section, we shall develop the picture fuzzy correlated geometric (PFCG) operator with picture fuzzy information based on the Choquet integral.

Definition 7. [37] Let $\alpha_j\,(j = 1, 2, \cdots, n)$ be a collection of PFNs. The picture fuzzy weighted geometric (PFWG) operator is a mapping $P^n \to P$ such that

$$\begin{aligned} &PFWG_\omega(\alpha_1, \alpha_2, \cdots, \alpha_n) \\ &= \bigotimes_{j=1}^{n} (\alpha_j)^{\omega_j} \\ &= \left(\prod_{j=1}^{n} (\mu_{\alpha_j})^{\omega_j}, 1 - \prod_{j=1}^{n}(1-\eta_{\alpha_j})^{\omega_j},\right. \\ &\left. 1 - \prod_{j=1}^{n} (1-\nu_{\alpha_j})^{\omega_j}\right) \end{aligned} \tag{4}$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be the weight vector of $\alpha_j (j = 1, 2, \cdots, n)$, and $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$.

Definition 8. [37] Let $\alpha_j (j = 1, 2, \cdots, n)$ be a collection of PFNs, the picture fuzzy ordered weighted geometric (PFOWG) operator of dimension n is a mapping PFOWG: $P^n \to P$, that has an associated weight vector $w = (w_1, w_2, \cdots, w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore,

$$\begin{aligned} &PFOWG_w(\alpha_1, \alpha_2, \cdots, \alpha_n) \\ &= \bigotimes_{j=1}^{n} (\alpha_{\sigma(j)})^{w_j} \\ &= \left(\prod_{j=1}^{n} (\mu_{\alpha_{\sigma(j)}})^{w_j}, 1 - \prod_{j=1}^{n}(1-\eta_{\alpha_{\sigma(j)}})^{w_j},\right. \\ &\left. 1 - \prod_{j=1}^{n} (1-\nu_{\alpha_{\sigma(j)}})^{w_j}\right) \end{aligned} \tag{5}$$

where $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, such that $\alpha_{\sigma(j-1)} \geq \alpha_{\sigma(j)}$ for all $j = 2, \cdots, n$.

Let $\mu(x_i)(i = 1, 2, \cdots, n)$ be the weight of the elements $x_i \in X(i = 1, 2, \cdots, n)$, where μ is a fuzzy measure, defined as follows:

Definition 9. [44] A fuzzy measure μ on the set X is a set function $\mu : \theta(X) \to [0, 1]$ satisfying the following axioms and $\theta(X)$ is the set of all subsets of X:

(1) $\mu(\phi) = 0, \mu(X) = 1$;
(2) $A \subseteq B$ implies $\mu(A) \leq \mu(B)$, for all $A, B \subseteq X$;
(3) $\mu(A \cup B) = \mu(A) + \mu(B) + \rho\mu(A)\mu(B)$, for all $A, B \subseteq X$ and $A \cap B = \phi$, where $\rho \in (-1, \infty)$.

Especially, if $\rho = 0$, then the condition (3) reduces to the axiom of additive measure: $\mu(A \cup B) = \mu(A) + \mu(B)$, for all $A, B \subseteq X$ and $A \cap B = \phi$.

If all the elements in X are independent, and we have $\mu(A) = \sum_{x_i \in A} \mu(\{x_i\})$, for all $A \subseteq X$.

Definition 10. [45] Let f be a positive real-valued function on X, and μ be a fuzzy measure on X. The discrete Choquet integral of f with respective to μ is defined by

$$C_\mu(f) = \sum_{i=1}^{n} f_{\sigma(i)} \left[\mu\left(A_{\sigma(i)}\right) - \mu\left(A_{\sigma(i-1)}\right)\right] \tag{6}$$

where $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, such that $f_{\sigma(i-1)} \geq f_{\sigma(i)}$ for all $j =$

$2, \cdots, n$, $A_{\sigma(k)} = \{x_{\sigma(j)} | j \le k.\}$, for $k \ge 1$, and $A_{\sigma(0)} = \phi$.

Based on Definition 10, In what follows, we shall develop the picture fuzzy correlated geometric (PFCG) operator based on the well-known Choquet integral [46–52] and geometric mean [53–63].

Definition 11. Let $\alpha_j = (\mu_j, \eta_j, \nu_j)(j = 1, 2, \cdots, n)$ be a collection of PFNs, and μ be a fuzzy measure on X, then we call then we define the picture fuzzy correlated geometric (PFCG) operator as follows:

$$\mathrm{PFCG}_\mu(a_1, a_2, \cdots, a_n) = \bigotimes_{j=1}^{n} \left(a_{\sigma(j)}\right)^{\left(\mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)\right)} \tag{7}$$

where $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, such that $a_{\sigma(j-1)} \ge a_{\sigma(j)}$ for all $j = 2, \cdots, n$, $A_{\sigma(k)} = \left\{x_{\sigma(j)} | j \le k\right\}$, for $k \ge 1$, and$A_{\sigma(0)} = \phi$.

Based on the operations of the picture fuzzy sets described, we can drive the Theorem 1.

Theorem 1. *Let $\alpha_j = \left(\mu_j, \eta_j, \nu_j\right)(j = 1, 2, \cdots, n)$ be a collection of PFNs, and μ be a fuzzy measure on X, then we call then we define the picture fuzzy correlated geometric (PFCG) operator as follows:*

$$\begin{aligned} &\mathrm{PFCG}_\mu(a_1, a_2, \cdots, a_n) \\ &= \bigotimes_{j=1}^{n} \left(a_{\sigma(j)}\right)^{\left(\mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)\right)} \\ &= \begin{pmatrix} \prod_{j=1}^{n} \left(\mu_{\alpha_{\sigma(j)}}\right)^{\left(\mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)\right)}, \\ 1 - \prod_{j=1}^{n} \left(1 - \eta_{\alpha_{\sigma(j)}}\right)^{\left(\mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)\right)}, \\ 1 - \prod_{j=1}^{n} \left(1 - \nu_{\alpha_{\sigma(j)}}\right)^{\left(\mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)\right)} \end{pmatrix} \end{aligned} \tag{8}$$

where $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, such that $a_{\sigma(j-1)} \ge a_{\sigma(j)}$ for all $j = 2, \cdots, n$, $A_{\sigma(k)} = \left\{x_{\sigma(j)} | j \le k\right\}$, for $k \ge 1$, and$A_{\sigma(0)} = \phi$.

Now we consider some special cases of the PFIOWA operator:

1) If $\mu\left(\left\{x_{\sigma(j)}\right\}\right) = \mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)$ for all j, then the PFCG operator becomes the picture fuzzy weighted geometric (PFWG):

$$\begin{aligned} &\mathrm{PFCG}_\mu(a_1, a_2, \cdots, a_n) \\ &= \bigotimes_{j=1}^{n} \left(a_{\sigma(j)}\right)^{\left(\mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)\right)} \\ &= \begin{pmatrix} \prod_{j=1}^{n} \left(\mu_{\alpha_{\sigma(j)}}\right)^{\left(\mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)\right)}, \\ 1 - \prod_{j=1}^{n} \left(1 - \eta_{\alpha_{\sigma(j)}}\right)^{\left(\mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)\right)}, \\ 1 - \prod_{j=1}^{n} \left(1 - \nu_{\alpha_{\sigma(j)}}\right)^{\left(\mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)\right)} \end{pmatrix} \\ &= PFWG_\omega(\alpha_1, \alpha_2, \cdots, \alpha_n) \\ &= \begin{pmatrix} \prod_{j=1}^{n} \left(\mu_{\alpha_j}\right)^{\omega_j}, \\ 1 - \prod_{j=1}^{n} \left(1 - \eta_{\alpha_j}\right)^{\omega_j}, 1 - \prod_{j=1}^{n} \left(1 - \nu_{\alpha_j}\right)^{\omega_j} \end{pmatrix} \end{aligned} \tag{9}$$

If $\mu(A) = \sum_{x_j \in A} \mu\left(\left\{x_j\right\}\right)$, for all$A \subseteq X$, where $|A|$ is the number of the elements in the set A, then $w_j = \mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)$, $i = 1, 2, \cdots, n$, where $w = (w_1, w_2, \cdots, w_n)^T$, $w_j \ge 0$, $i = 1, 2, \cdots, n$, and $\sum_{j=1}^{n} w_j = 1$, then the PFCG operator becomes the picture fuzzy ordered weighted geometric (PFOWG) operator:

$$\begin{aligned} &\mathrm{PFCG}_\mu(a_1, a_2, \cdots, a_n) \\ &= \bigotimes_{j=1}^{n} \left(a_{\sigma(j)}\right)^{\left(\mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)\right)} \\ &= \begin{pmatrix} \prod_{j=1}^{n} \left(\mu_{\alpha_{\sigma(j)}}\right)^{\left(\mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)\right)}, \\ 1 - \prod_{j=1}^{n} \left(1 - \eta_{\alpha_{\sigma(j)}}\right)^{\left(\mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)\right)}, \\ 1 - \prod_{j=1}^{n} \left(1 - \nu_{\alpha_{\sigma(j)}}\right)^{\left(\mu\left(A_{\sigma(j)}\right) - \mu\left(A_{\sigma(j-1)}\right)\right)} \end{pmatrix} \\ &= PFOWG_w(\alpha_1, \alpha_2, \cdots, \alpha_n) \\ &= \begin{pmatrix} \prod_{j=1}^{n} \left(\mu_{\alpha_{\sigma(j)}}\right)^{w_j}, \\ 1 - \prod_{j=1}^{n} \left(1 - \eta_{\alpha_{\sigma(j)}}\right)^{w_j}, 1 - \prod_{j=1}^{n} \left(1 - \nu_{\alpha_{\sigma(j)}}\right)^{w_j} \end{pmatrix} \end{aligned} \tag{10}$$

It can be easily proved that the PFCG operator has the following properties.

Property 1. *(Idempotency) If all $\alpha_j\,(j = 1, 2, \cdots, n)$ are equal, i.e. $\alpha_j = \alpha$ for all j, then*

$$\mathrm{PFCG}_{\mu}\left(a_1, a_2, \cdots, a_n\right) = \alpha \tag{11}$$

Property 2. *(Boundedness) Let $\alpha_j\,(j = 1, 2, \cdots, n)$ be a collection of PFNs, and let*

$$\alpha^- = \min_j \alpha_j, \alpha^+ = \max_j \alpha_j$$

Then

$$\alpha^- \leq \mathrm{PFCG}_{\mu}\left(a_1, a_2, \cdots, a_n\right) \leq \alpha^+ \tag{12}$$

Property 3. *(Monotonicity) Let $\alpha_j\,(j = 1, 2, \cdots, n)$ and $\alpha'_j\,(j = 1, 2, \cdots, n)$ be two set of PFNs, if $\alpha_j \leq \alpha'_j$, for all j, then*

$$\begin{aligned}&\mathrm{PFCG}_{\mu}\left(a_1, a_2, \cdots, a_n\right)\\ &\leq \mathrm{PFCG}_{\mu}\left(a'_1, a'_2, \cdots, a'_n\right)\end{aligned} \tag{13}$$

Property 4. *(Commutativity) Let $\alpha_j (j = 1, 2, \cdots, n)$ be a set of PFNs, if $\alpha_j \leq \alpha'_j$, for all j, then*

$$\begin{aligned}&\mathrm{PFCG}_{\mu}\left(a_1, a_2, \cdots, a_n\right)\\ &= \mathrm{PFCG}_{\mu}\left(a'_1, a'_2, \cdots, a'_n\right)\end{aligned} \tag{14}$$

where $\alpha'_j\,(j = 1, 2, \cdots, n)$ is any permutation of $\alpha_j\,(j = 1, 2, \cdots, n)$.

4. Models for MADM with picture fuzzy information

Based the PFCG operator, in this section, we shall study the MADM with picture fuzzy information. Let $A = \{A_1, A_2, \cdots, A_m\}$ be a discrete set of alternatives, and $G = \{G_1, G_2, \cdots, G_n\}$ be the set of attributes. Suppose that$R = \left(r_{ij}\right)_{m\times n} = \left(\mu_{ij}, \eta_{ij}, \nu_{ij}\right)_{m\times n}$ is the picture fuzzy decision matrix, where μ_{ij} indicates the degree of positive membership that the alternative A_i satisfies the attribute G_j given by the decision maker, η_{ij} indicates the degree of neutral membership that the alternative A_i doesn't satisfy the attribute G_j, ν_{ij} indicates the degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker, $\mu_{ij} \in [0, 1]$, $\eta_{ij} \in [0, 1]$ $\nu_{ij} \in [0, 1]$, $\mu_{ij} + \eta_{ij} + \nu_{ij} \leq 1$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$.

In the following, we apply the PFCG operator to the MADM problems with picture fuzzy information.

Step 1. We utilize the decision information matrix R, and the PFCG operator

$$\begin{aligned}\alpha_i &= \mathrm{PFCG}_{\mu}\left(r_{i1}, r_{i2}, \cdots, r_{in}\right)\\ &= \bigotimes_{j=1}^{n} \left(r_{\sigma(ij)}\right)^{\left(\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)\right)}\\ &\left(\begin{array}{l}\prod_{j=1}^{n} \left(\mu_{\alpha_{\sigma(ij)}}\right)^{\left(\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)\right)}\\ 1-\prod_{j=1}^{n}\left(1-\eta_{\alpha_{\sigma(ij)}}\right)^{\left(\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)\right)}\\ 1-\prod_{j=1}^{n}\left(1-\nu_{\alpha_{\sigma(ij)}}\right)^{\left(\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)\right)}\end{array}\right)\\ &i = 1, 2, \cdots, m.\end{aligned} \tag{15}$$

to derive the overall values $\alpha_i\,(i = 1, 2, \cdots, m)$ of the alternative A_i.

Step 2. Calculate the scores $S\left(\alpha_i\right)$ $(i = 1, 2, \cdots, m)$ of the overall picture fuzzy numbers $\alpha_i\,(i = 1, 2, \cdots, m)$ to rank all the alternatives $A_i\,(i = 1, 2, \cdots, m)$ and then to select the best one(s).

Step 3. Rank all the alternatives $A_i\,(i = 1, 2, \cdots, m)$ and select the best one(s) in accordance with $S\left(\alpha_i\right)$ $(i = 1, 2, \cdots, m)$.

5. Numerical example

Execution of public policy is public authority enforces the route of the party and the state of policies, laws and regulations and decrees instructions, superior decision command ability, is the public authority departments through the process, the decision-making process and running process to ensure that the ability to set goals. Customs is the national inbound and outbound means of supervision and administration authority, is the forefront of the national foreign trade and the window, shoulders the important mission of security doors, as the country one of the important public policy enforcement agencies, the strength of the policy executive power of the image directly affects the country opening to the outside world, directly affects the development of the open economy in our country. As perform the governance of the state administrative organs, and the

Table 1
The picture fuzzy decision matrix

	A_1	A_2	A_3	A_4	A_5
G_1	(0.03,0.81,0.13)	(0.04,0.83,0.10)	(0.02,0.82,0.05)	(0.05,0.85,0.06)	(0.42,0.34,0.18)
G_2	(0.71,0.15,0.08)	(0.68,0.26,0.05)	(0.08,0.84,0.06)	(0.13,0.75,0.09)	(0.08,0.86,0.02)
G_3	(0.70,0.12,0.08)	(0.90,0.03,0.02)	(0.83,0.09,0.05)	(0.90,0.06,0.02)	(0.54,0.32,0.09)
G_4	(0.13,0.62,0.21)	(0.07,0.19,0.05)	(0.73,0.15,0.10)	(0.67,0.09,0.21)	(0.86,0.08,0.03)

customs in the process of administration according to law should avoid policy implementation deviation, effective control of difference, the maximum to keep the public policy implementation unification principle and fair law enforcement, however, in the current customs law enforcement practice, the uniformity problem became former customs enforcement in universal existence, society, reflects the problems of the relatively strong. The public policy execution ability evaluation is classical MADM problem [64–72]. In this section, we utilize a practical MADM problem for evaluating the public policy execution ability. Suppose an organization plans to evaluate the public policy execution ability. By collecting all possible information about public policy execution ability, project term choose five potential public policy departments $A_i\,(i = 1, 2, \cdots, 5)$ as candidates. The project team selects four attributes to evaluate the public policy execution ability of five potential public policy departments: ①G_1 is the consensus on policy objectives; ②G_2 is the ability to learn and grow; ③G_3 is the policy implementation process; ④G_4 is the policy implementation results. The five possible potential public policy departments$A_i\,(i = 1, 2, \cdots, 5)$ are to be evaluated using the picture fuzzy numbers by the decision makers under the above four attributes, and construct the following matrix $\tilde{R} = \left(\tilde{r}_{ij}\right)_{5\times4}$ is shown in Table 1.

In the following, in order to select the most desirable public policy departments, we utilize the PFCG operator to multiple attribute decision making problems with picture fuzzy information, which can be described as following.

Step 1. Suppose the fuzzy measure of attribute of $G_j\,(j = 1, 2, \cdots, n)$ and attribute sets of G as follows:

$$\mu(G_1) = 0.35, \mu(G_2) = 0.30, \mu(G_3) = 0.25,$$

$$\mu(G_4) = 0.20, \mu(G_1, G_2) = 0.60, \mu(G_1, G_3) = 0.55$$

$$\mu(G_1, G_4) = 0.50, \mu(G_2, G_3) = 0.65, \mu(G_2, G_4) = 0.50$$

$$\mu(G_3, G_4) = 0.45, \mu(G_1, G_2, G_3) = 0.80,$$

$$\mu(G_1, G_2, G_4) = 0.85,$$

$$\mu(G_1, G_3, G_4) = 0.70, \mu(G_2, G_3, G_4) = 0.75,$$

$$\mu(G_1, G_2, G_3, G_4) = 1.00$$

Step 2. Utilize the decision information given in matrix $\tilde{R}$, and the PFCG operator, we obtain the overall values $\tilde{r}_i$ of the public policy departments $A_i\,(i = 1, 2, \cdots, 5)$.

$$\tilde{r}_1 = (0.429,0.398,0.104)$$

$$\tilde{r}_2 = (0.412,0.464,0.027)$$

$$\tilde{r}_3 = (0.544,0.278,0.163)$$

$$\tilde{r}_4 = (0.626,0.229,0.113)$$

$$\tilde{r}_5 = (0.449,0.413,0.026)$$

Step 3. Calculate the scores $S(\tilde{r}_i)\,(i = 1, 2, \cdots, 5)$ of the overall picture fuzzy values $\tilde{r}_i\,(i = 1, 2, \cdots, 5)$

$$S(\tilde{r}_1) = -0.129, S(\tilde{r}_2) = 0.078$$

$$S(\tilde{r}_3) = -0.219, S(\tilde{r}_4) = 0.035$$

$$S(\tilde{r}_5) = 0.173$$

Step 4. Rank all the public policy departments $A_i(i = 1, 2, 3, 4, 5)$ in accordance with the scores $S(\tilde{r}_i)\ \ (i = 1, 2, \cdots, 5)$ of the overall preference values $\tilde{r}_i\,(i = 1, 2, \cdots, 5)$: $A_5 \succ A_2 \succ A_4 \succ A_1 \succ A_3$, and thus the most desirable public policy department is A_5.

6. Conclusion

In this paper, we investigate the multiple attribute decision making problems with picture fuzzy information. Then, we utilize Choquet integral to propose the picture fuzzy correlated geometric (PFCG) operator. The prominent characteristic of this proposed operator are studied. Then, we have utilized the picture fuzzy correlated geometric (PFCG) operator to deal with the picture fuzzy multiple attribute decision making problems. Finally, a practical example for evaluating the public policy execution ability is given to verify the developed approach. In the future, the application of the proposed aggregating operators of PFSs needs to be explored in the other many uncertain environments [73–82].

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